ShapeCoefficients := 4

Frequency := 0 .. ShapeCoefficients

$\text{ShapeCoefficient}_{\text{Frequency}} :=$

1.0
1.1913785723
−0.0793018558
−0.2171442026
0.0014526076

$$\text{PulseShape(t)} := \sum_{\text{Frequency} = 0}^{\text{ShapeCoefficients}} \left(\text{ShapeCoefficient}_{\text{Frequency}} \cdot \cos(2 \cdot \pi \cdot \text{Frequency} \cdot t) \right)$$

Time := −0.5, −0.48 .. 0.5
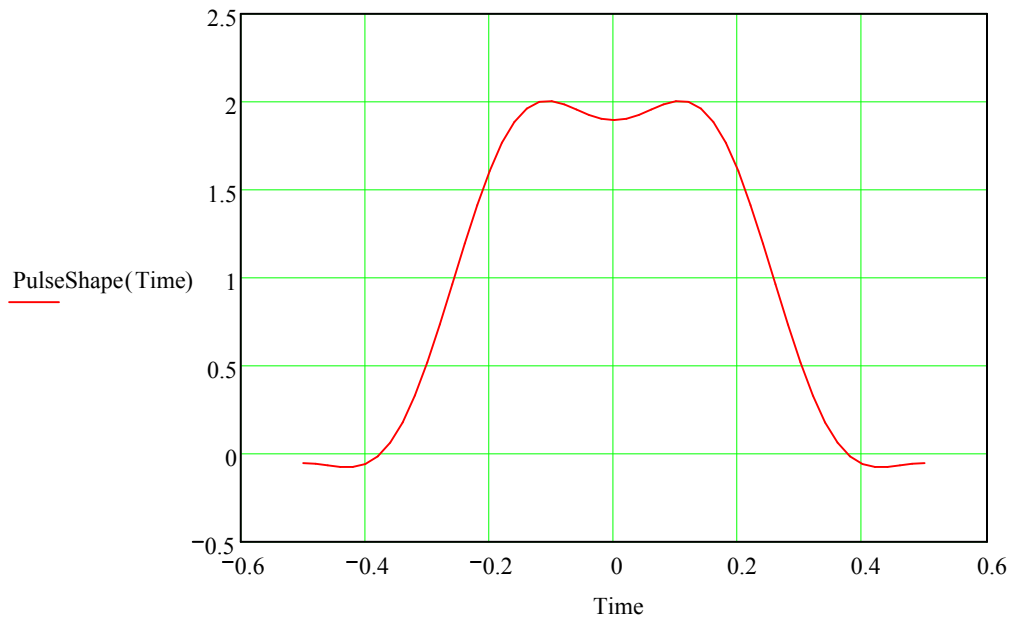
$$\text{WhiteNoise}(\sigma) := \sigma \cdot \sqrt{-2 \cdot \ln(\text{rnd}(1.0))} \cdot \cos(\text{rnd}(2 \cdot \pi))$$

SampleRate := 8192

Bandwidth := 2048

Tones := 128

LowerBandEdge := 1000

$$\text{SymbolSeparation} := \frac{\text{SampleRate}}{\dfrac{\text{Bandwidth}}{\text{Tones}}}$$

SymbolSeparation = 512

$$\text{SymbolsPerSecond} := \frac{\text{SampleRate}}{\text{SymbolSeparation}}$$

BitsPerSecond := SymbolsPerSecond·log(Tones, 2)

ShapeLength := 2·SymbolSeparation

$$\text{ShapeLength} = 1.024 \times 10^3$$

$$\text{Symbol}(\text{Tone}, \text{Time}, \phi) := \left| \begin{array}{l} \text{ToneFrequency} \leftarrow \dfrac{\text{LowerBandEdge}}{\text{SampleRate}} + \dfrac{\text{Tone}}{\text{Tones}} \cdot \dfrac{\text{Bandwidth}}{\text{SampleRate}} \\[2ex] 0 \quad \text{if} \quad \text{Time} < \dfrac{-\text{ShapeLength}}{2} \\[2ex] 0 \quad \text{if} \quad \text{Time} \geq \dfrac{\text{ShapeLength}}{2} \\[2ex] \text{PulseShape}\left(\dfrac{\text{Time}}{\text{ShapeLength}}\right) \cdot \cos(2 \cdot \pi \cdot \text{Time} \cdot \text{ToneFrequency} + \phi) \quad \text{otherwise} \end{array} \right.$$
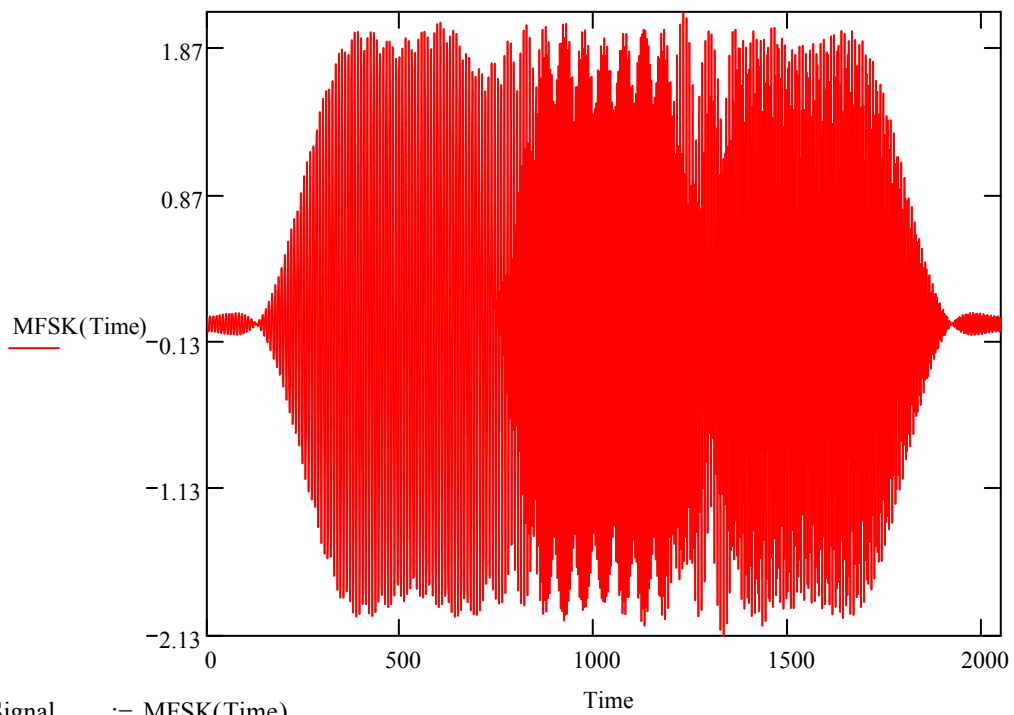
Time := −ShapeLength .. ShapeLength
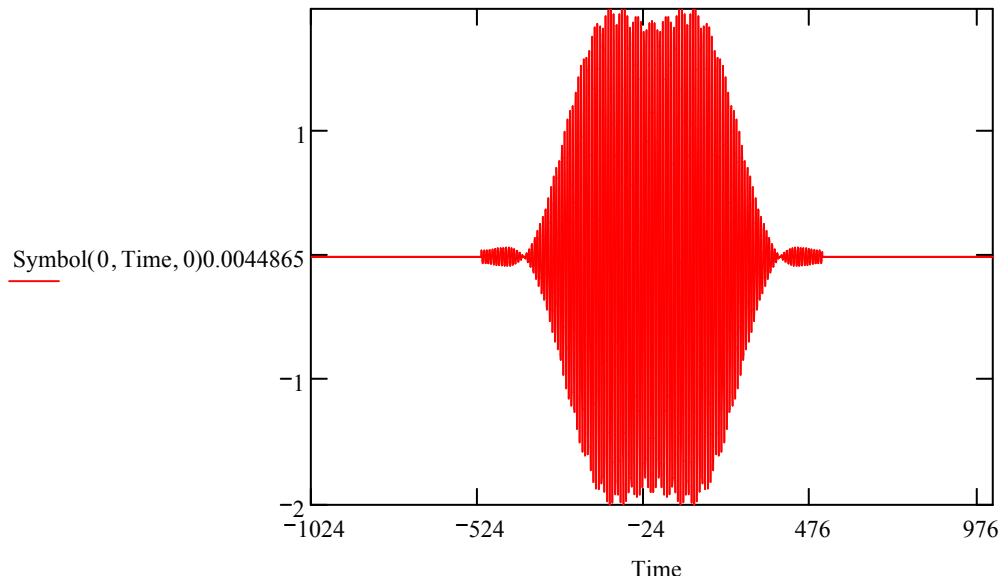
$$\text{MFSK(Time)} := \text{Symbol}(0, \text{Time} - \text{SymbolSeparation}, 0) + \text{Symbol}(63, \text{Time} - 2 \cdot \text{SymbolSeparation}, 0) +$$

$$\text{Noisy\_MFSK(Time)} := \text{MFSK(Time)} + \text{WhiteNoise}(3)$$

$$\text{Time} := 0 .. 4 \cdot \text{SymbolSeparation} - 1$$

$$\text{Signal}_{\text{Time}} := \text{MFSK(Time)}$$

$$\text{NoisySignal}_{\text{Time}} := \text{Noisy\_MFSK(Time)}$$

$$\text{cumulative\_rft}(\text{signal}, N, K\_Max) := \begin{array}{|l} \text{cos\_offset} \leftarrow \dfrac{N}{4} \\[2ex] \text{for } i \in 0 \, .. \, N - 1 \\ \quad \begin{array}{|l} \text{lut}_i \leftarrow \dfrac{\sin\left(i \cdot 2 \cdot \dfrac{\pi}{N}\right)}{N} \\[3ex] \text{fifo}_i \leftarrow 0 \end{array} \\[4ex] \text{fifo\_index} \leftarrow 0 \\[1ex] \text{for } k \in 0 \, .. \, K\_Max \\ \quad \begin{array}{|l} a_k \leftarrow 0 \\[1ex] b_k \leftarrow 0 \end{array} \\[3ex] \text{for } \text{sample} \in 0 \, .. \, \text{length}(\text{signal}) - 1 \\ \quad \begin{array}{|l} \text{old} \leftarrow \text{fifo}_{\text{fifo\_index}} \\[1ex] \text{fifo}_{\text{fifo\_index}} \leftarrow \text{signal}_{\text{sample}} \\[1ex] \text{diff} \leftarrow \text{Signal}_{\text{sample}} - \text{old} \\[1ex] a_0 \leftarrow a_0 + \text{diff} \cdot \text{lut}_{\text{cos\_offset}} \\[1ex] \text{for } k \in 1 \, .. \, K\_Max \\ \quad \begin{array}{|l} j \leftarrow \text{mod}(k \cdot \text{fifo\_index}, N) \\[1ex] q \leftarrow \text{mod}(j + \text{cos\_offset}, N) \\[1ex] a_k \leftarrow a_k + \text{diff} \cdot \text{lut}_q \\[1ex] b_k \leftarrow b_k + \text{diff} \cdot \text{lut}_j \end{array} \\[3ex] \text{for } k \in 0 \, .. \, K\_Max \\ \quad \text{mag}_k \leftarrow \sqrt{\left(a_k\right)^2 + \left(b_k\right)^2} \\[2ex] \text{psd}^{\langle \text{sample} \rangle} \leftarrow \text{mag} \\[1ex] \text{fifo\_index} \leftarrow \text{mod}(\text{fifo\_index} + 1, N) \end{array} \\[2ex] \text{psd} \end{array}$$
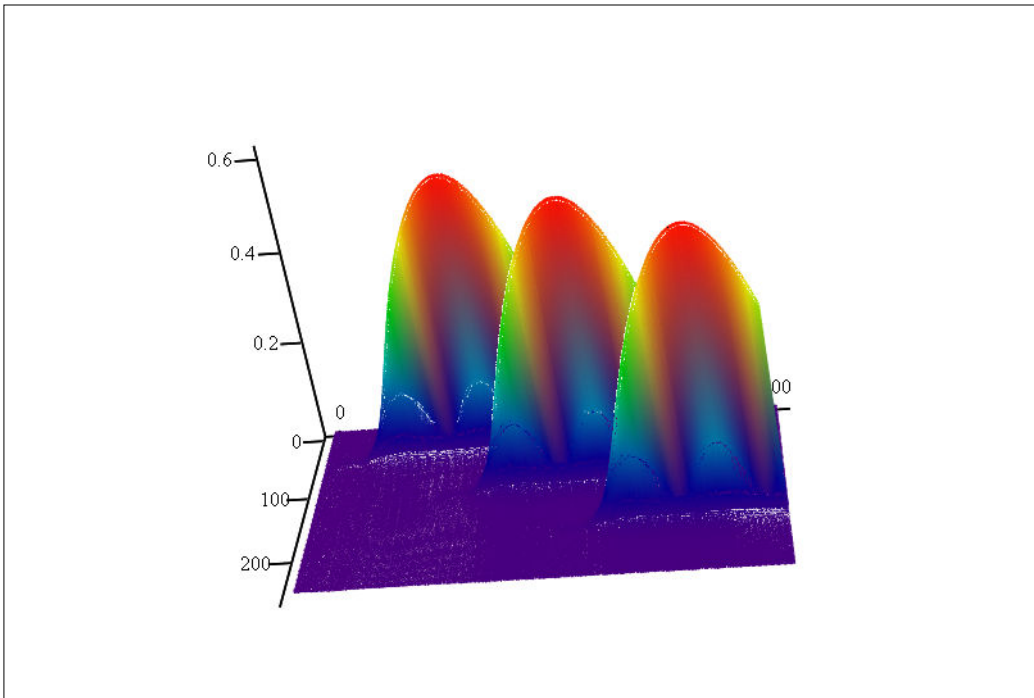
$$\text{Frequency}(\text{harmonic}) := \text{harmonic} \cdot \frac{\text{SampleRate}}{2 \cdot \text{ShapeLength}}$$

$$\text{cumulative\_rft\_spectrum} := \text{cumulative\_rft}\left(\text{Signal}, \text{SymbolSeparation}, \frac{\text{SymbolSeparation}}{2}\right)$$

$$\text{noisy\_cumulative\_rft\_spectrum} := \text{cumulative\_rft}\left(\text{NoisySignal}, \text{SymbolSeparation}, \frac{\text{SymbolSeparation}}{2}\right)$$
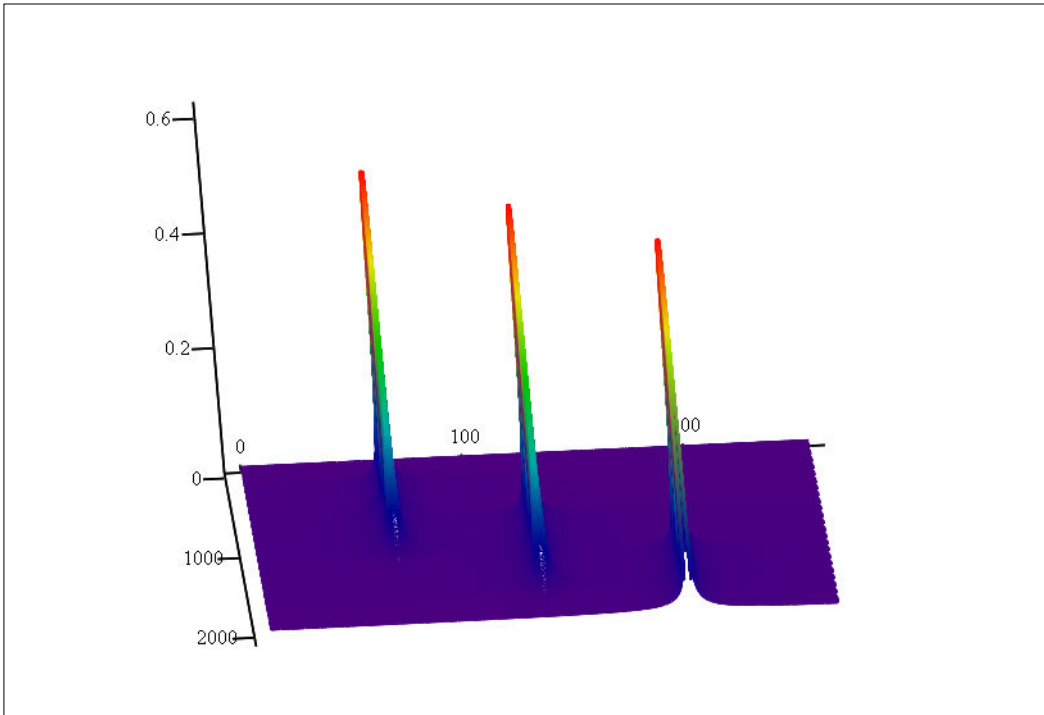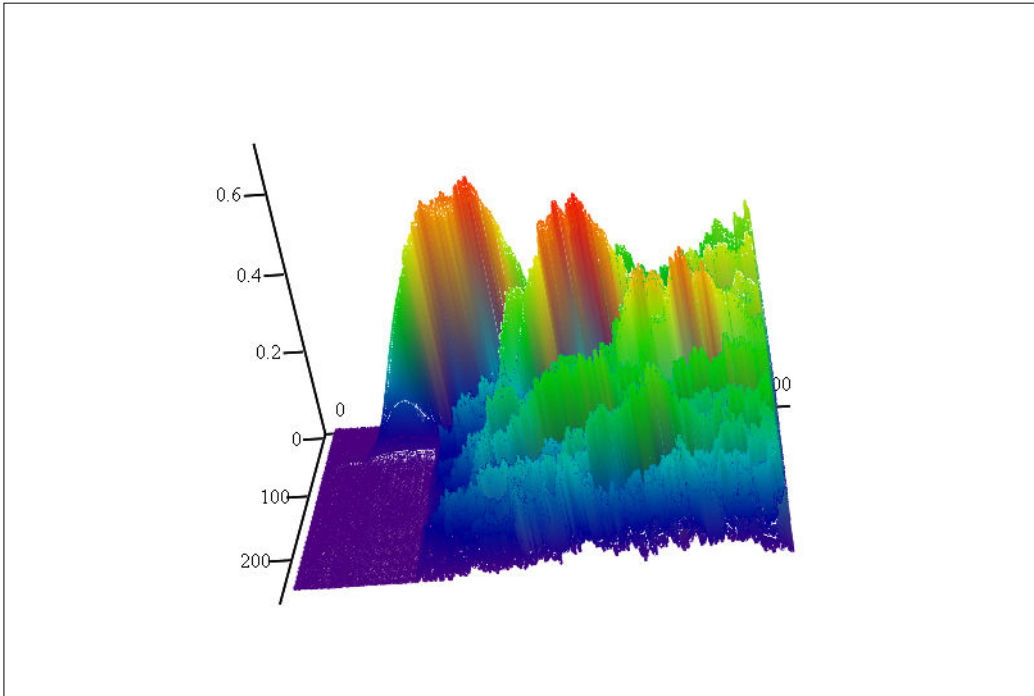
cumulative_rft_spectrum^T

cumulative_rft_spectrum
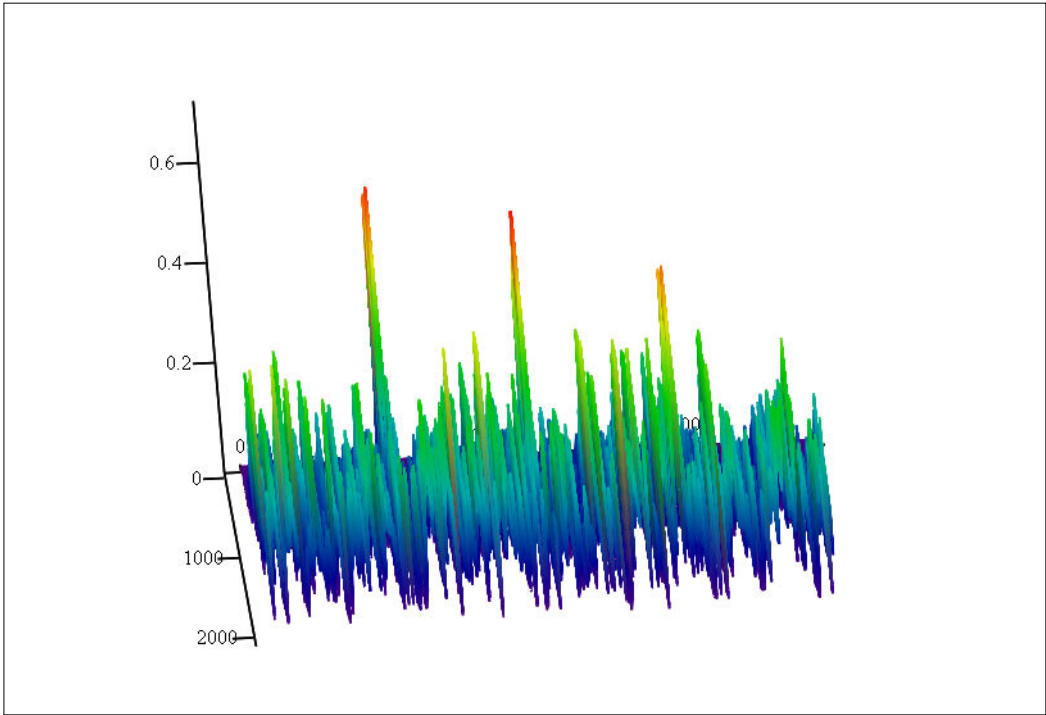
noisy_cumulative_rft_spectrum^T

noisy_cumulative_rft_spectrum